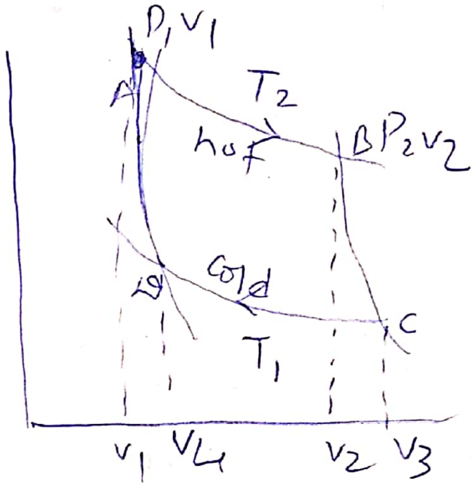
TDC-II (Hon.) & Sub. / Paper-IV (for Hons.)

Carnot Cycle and its efficiency

Carnot employed a reversible cycle to demonstrate the maximum convertibility of heat into work. The system consisting of 1 mole of an ideal gas and is subjected to four successive operations

(I) Isothermal Expansion (AB)
When gas is allowed to expand reversibly & isothermally at temperature T_2 from volume V_1 to V_2



$\Delta T = 0$, $\Delta U = 0$

$$q_2 = -w_1 = RT\ln\frac{V_2}{V_1} \longrightarrow (I)$$

(II) Adiabatic Expansion (BC) → BC path indicates, that gas is allowed to expand from V_2 to V_3 adiabatically & reversibly

$$q = 0$$

$$\Delta U = q + w = w$$

$$\Delta U = C_v(T_1 - T_2) = -ve$$

$$-w_2 = -C_v(T_2 - T_1) \longrightarrow (II)$$

(III) Isothermal Compression (CD) ⇒ When Gas further undergoes isothermal Compression at Temperature (T_1) from volume V_3 to V_4

$$\Delta T_{CD} = 0, \quad \Delta U_{CD} = 0$$

$$-q_1 = W_3 = RT \ln \frac{V_4}{V_3}$$

$$q_1 = -RT \ln \frac{V_4}{V_3} \quad \text{—— (III)}$$

(IV) Adiabatic Compression ⇒ In final stage adiabatic & reversible compression takes place from V_4 to V_1

$$q_{DA} = 0$$

$$\Delta U_{DA} = W_{DA} = C_V (T_2 - T_1) = +ve.$$

$$W_4 = C_V (T_2 - T_1) \quad \text{—— (IV)}$$

Net heat absorbed (q) in whole cycle

$$= q_2 + (-q_1) = RT_2 \left(\frac{V_2}{V_1}\right) + RT_1 \ln \frac{V_4}{V_3}$$

$$= RT_2 \left(\frac{V_2}{V_1}\right) - RT_1 \ln \frac{V_3}{V_4}$$

$$= R(T_2 - T_1) \ln \frac{V_2}{V_1} \quad \left[\because \frac{V_2}{V_1} = \frac{V_3}{V_4}\right]$$

Net work done by the Gas in whole Cycle

$$W = -W_1 - W_2 + W_3 + W_4$$

$$= RT \ln \frac{V_2}{V_1} - C_V(T_2 - T_1) + RT_1 \left(\frac{V_4}{V_3}\right) + C_V(T_2 - T_1)$$

$$= RT_2 \ln \frac{V_2}{V_1} - RT_1 \ln \frac{V_3}{V_4}$$

$$= R(T_2 - T_1) \ln \frac{V_2}{V_1}$$

Thus net condition for cyclic process is that the net work done is equal to the heat absorbed is fully satisfied.

Efficiency of carnot engine = It is defined as the fraction of the heat absorbed by an engine which it can convert into work.

$$\text{efficiency } (\eta) = \frac{W}{Q_2} = \frac{R(T_2 - T_1) \ln \frac{V_2}{V_1}}{RT_2 \ln \frac{V_2}{V_1}} = \frac{T_2 - T_1}{T_2}$$

η is less than unity hence no heat engine has been constructed with efficiency equal to unity.